TRINITY COLLEGE

Semester Two Examination, 2016

Question/Answer Booklet

MATHEMATICS APPLICATIONS UNITS 3 AND 4

Section One: Calculator-free

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Your name SOLUTIONS

Time allowed for this section

Reading time before commencing work: five minutes
Working time for section: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer Booklet
Formula Sheet

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised notes or other items of a non-personal nature in the examination room. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of exam
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	13	13	100	98	65
			Total	149	100

Instructions to candidates

1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer Booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Spare pages are included at the end of this booklet. They can be used for planning your responses and/or as additional space if required to continue an answer.
   - Planning: If you use the spare pages for planning, indicate this clearly at the top of the page.
   - Continuing an answer: If you need to use the space to continue an answer, indicate in the original answer space where the answer is continued, i.e. give the page number. Fill in the number of the question that you are continuing to answer at the top of the page.

5. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you **do not use pencil**, except in diagrams.

7. The Formula Sheet is **not** to be handed in with your Question/Answer Booklet.

See next page

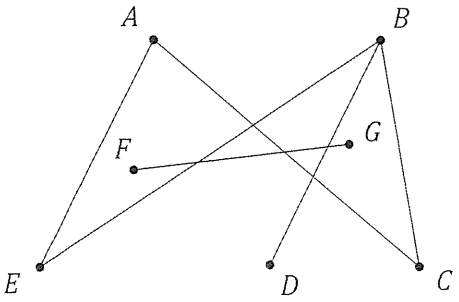
Section One: Calculator-free **35% (53 Marks)**

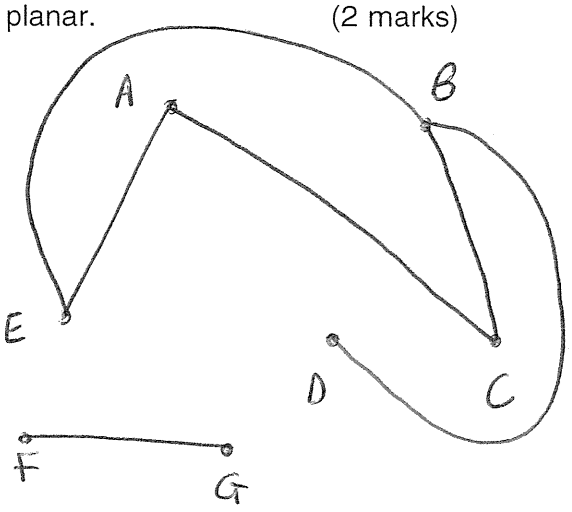
This section has **eight (8)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time for this section is 50 minutes.

Question 1 **(5 marks)**

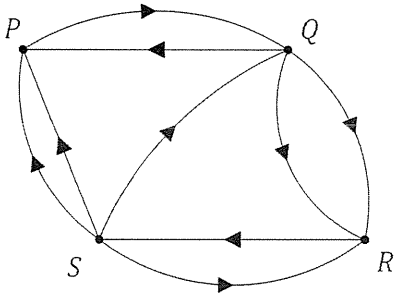
(a) Re-draw the following graph to clearly illustrate that it is planar. (2 marks)





no edges cross ✓
correct & disconnected ✓

(b) A digraph is shown below.



(i) Construct an adjacency matrix M from the digraph. (2 marks)

$$\begin{array}{c} \\ P \\ Q \\ R \\ S \end{array} \begin{array}{c} \begin{array}{cccc} P & Q & R & S \end{array} \\ \begin{bmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 2 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

✓✓

(ii) Explain what information the matrix M^2 would show. (1 mark)

The number of 2 stage paths ✓ between vertices

Question 2 **(8 marks)**

The number of laptop computers, T_n, that were brought to a school IT department for recharging during week n of the school year can be described recursively by the rule

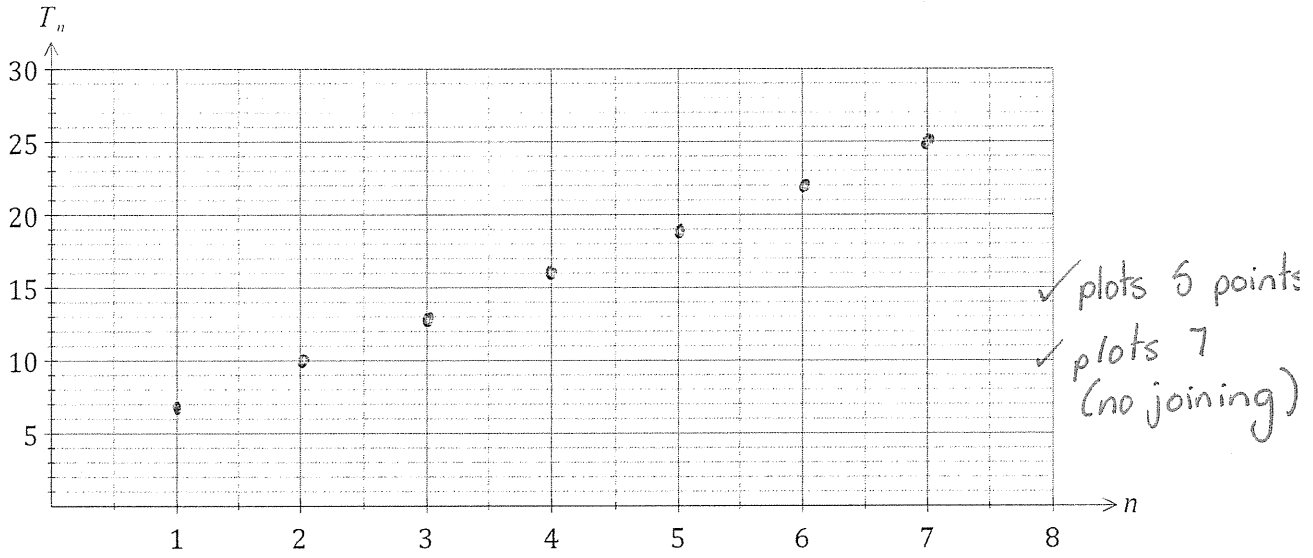
$$T_{n+1} = T_n + 3, \qquad T_4 = 16$$

(a) Use the rule to complete the table below. (2 marks)

n	1	2	3	4	5	6	7
T_n	7	10	13	16	19	22	25

✓ ✓

(b) Display the terms of the sequence from the table on the graph below. (2 marks)



(c) A rule to determine the number of laptops brought for recharging during week n can also be written in the form $T_n = an + b$. Determine the values of a and b. (2 marks)

$T_n = 7 + (n-1) \times 3$

$= 7 + 3n - 3$

$T_n = 4 + 3n$ ✓

$a = 3, \; b = 4$ ✓

(d) If the pattern continued, determine the number of the week during which the number of laptops brought in for recharging first exceeds 50. (2 marks)

$50 = 4 + 3n$ ✓

$46 = 3n$

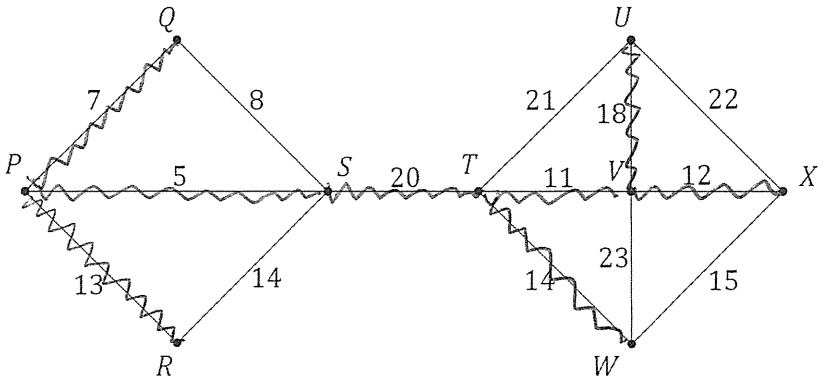
$n = 15.\dot{3}$

$\therefore$ Week 16 ✓

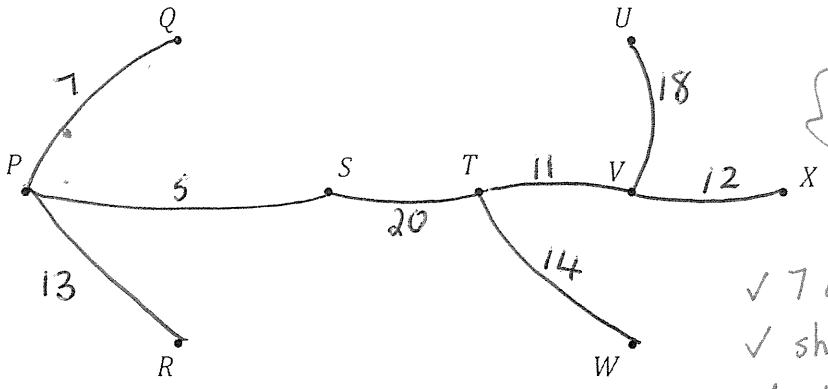
$3 \overline{)46.0}$ 15.3

Question 3 **(5 marks)**

The network below shows the distances, in kilometres, between nine towns, P, Q, R, ..., X.



(a) Use Prim's algorithm, starting from P, to determine a minimum spanning tree for the network and draw the minimum spanning tree below. (3 marks)



numbers not needed

✓ 7 correct edges
✓ shows all
✓ draws tree, no cycles

(b) State the length of the minimum spanning tree. (1 mark)

100 Km ✓

(c) Prim's algorithm from different starting points produces the same minimum spanning tree for this network. State the final edge that would complete the minimum spanning tree using Prim's algorithm starting from U. (1 mark)

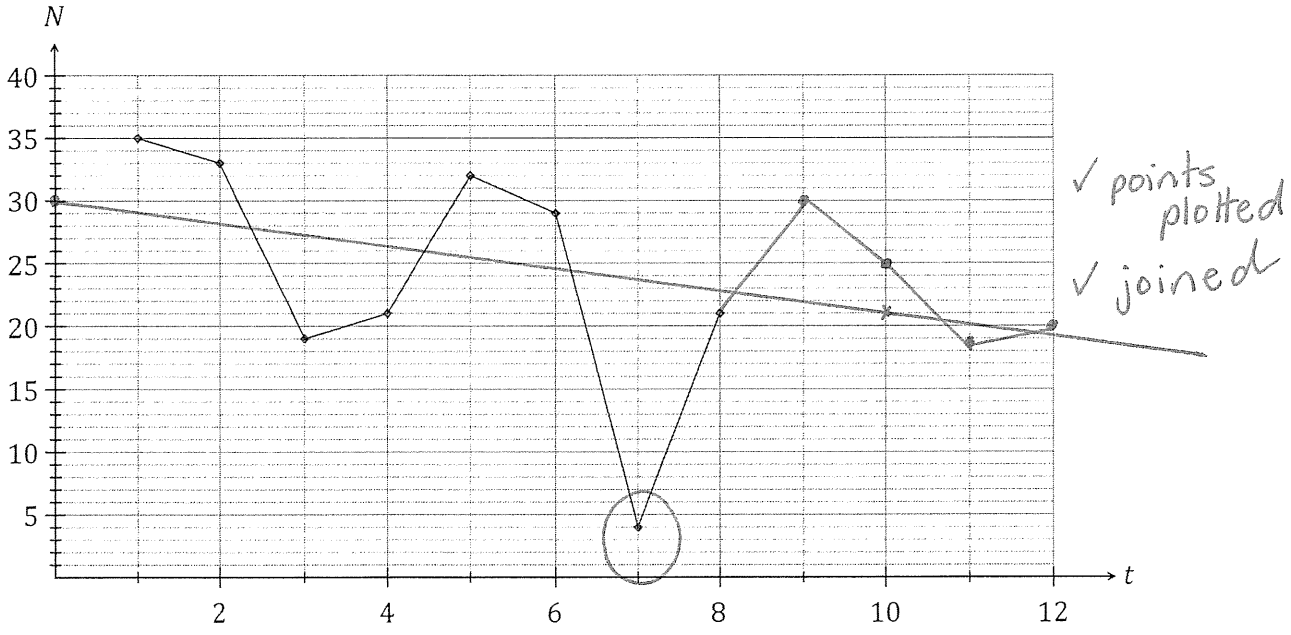
PR ✓

Question 4 **(8 marks)**

The table below shows the number of members, N, of a swimming club who turned up for training over the first three weeks of winter. The club trained on Monday, Tuesday, Wednesday and Thursday each week.

	Week 1				Week 2				Week 3			
Day	M	T	W	T	M	T	W	T	M	T	W	T
Time, t	1	2	3	4	5	6	7	8	9	10	11	12
Number, N	35	33	19	21	32	29	4	21	30	25	19	20

(a) Complete the time series plot of this data on the axes below. (2 marks)



(b) One score appears to be an outlier. Circle the score on the plot. ✓ (1 mark)

(c) Comment on the long term trend and seasonality of the time series plot. (2 marks)

Decreasing trend ✓
4 seasons in a cycle (4 day cycle) ✓

(d) The trend line for the data is $N = 30 - 0.9t$.

(i) Sketch the trend line on the axes above. (2 marks)

(10, 21)
✓ y int is 30 ✓ another pt & line correct.

(ii) Comment on the number of swimmers at training on Wednesdays in relation to the trend line. (1 mark)

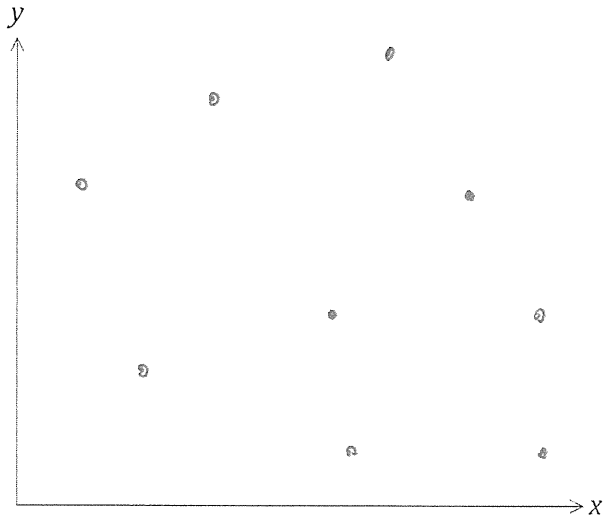
Wednesdays are below the trend line ✓
↑ must mention line.

See next page

Question 5 **(4 marks)**

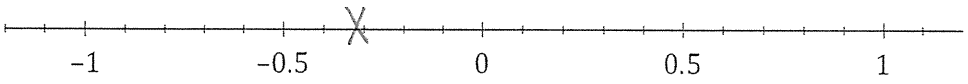
A student investigated the relationship between the number of emails, x, and the number of text messages, y, received by nine people during a weekday and found that there was a weak negative linear relationship between the variables.

(a) Use the axes below to sketch a possible scatter plot that reflects this information. (2 marks)



weak negative ✓
9 points ✓

(b) Place a cross on the scale below for the best estimate you can make for the value of the correlation coefficient between the two variables. (1 mark)



between −0.2 and −0.5

(c) Suggest a reason for the relationship the student found. (1 mark)

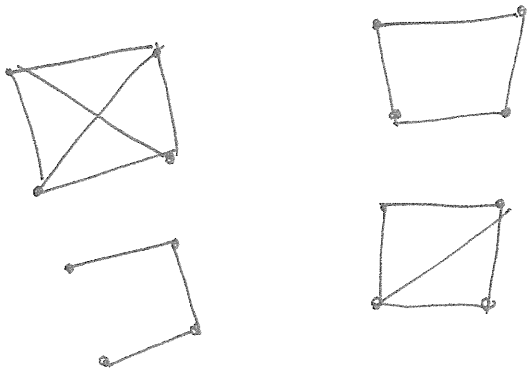
personal preference
texting easier
any valid reason

Question 6 **(8 marks)**

(a) A simple connected graph has 4 edges and V vertices. State all possible values of V. (2 marks)

V = 4 or V = 5

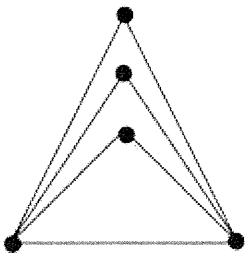
(b) A simple connected graph has E edges and 4 vertices. State all possible values of E. (2 marks)



E = 3, 4, 5, 6

✓ two correct
✓ all correct

(c) Consider the simple connected graph with 5 vertices drawn below.



must have

(i) Is the graph Eulerian? Explain your answer. (2 marks)

Yes ✓
Traversable and starts and finishes at same vertex ✓

(ii) Is the graph Hamiltonian? Explain your answer. (2 marks)

No ✓
The walk is not closed - semi Hamiltonian.
✓ each vertex can be visited once but starts and finishes at different place.

See next page

Question 7 **(7 marks)**

The table below shows all but one of the seasonal indices for the daily sales figures for a recently opened pop-up store that is closed on Mondays and Tuesdays.

Day	Wednesday	Thursday	Friday	Saturday	Sunday
Seasonal index	0.65	0.70		1.35	1.50

(a) Determine the seasonal index for Friday. (2 marks)

$0.65 + 0.7 + x + 1.35 + 1.5 = 5$ ✓

$4.2 + x = 5$

$x = 0.8$ ✓

(b) One weekend, the pop-up store had sales worth \$3 000 on Saturday and \$3 000 on Sunday. If these sales figures were deseasonalised, which day would have the highest sales? Justify your answer. (2 marks)

$3000 \div 1.35 > 3000 \div 1.5$ ✓

$\therefore$ Saturday has highest ✓

(c) The equation of the trend line for deseasonalised sales, S in dollars, after the store had been open for t days was $S = 195t + 1640$.

(i) Interpret the value of the slope of this line in this context. (1 mark)

Sales are increasing by \$195 per day ✓

(ii) Write down an expression that would forecast the actual sales of the pop-up shop on the Wednesday when $t = 31$. **Do not** evaluate your expression. (2 marks)

$(195 \times 31 + 1640) \times 0.65$

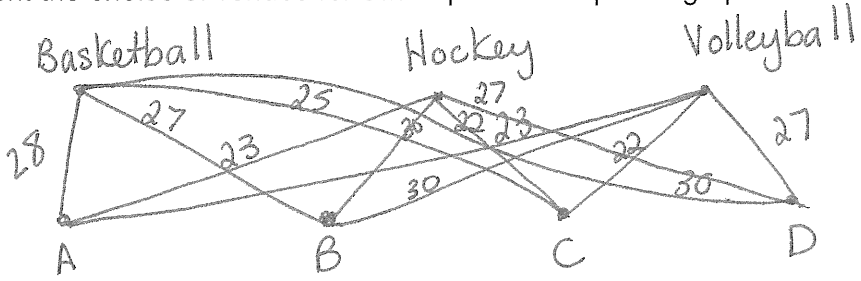
✓ ✓

Question 8 **(8 marks)**

A school has received quotes from venues A, B, C and D to host teams of students to play knockout competitions in basketball, hockey and volleyball. The four venues can only host one sport at a time and their quotes, in hundreds of dollars, for each sport are shown below.

		Sport		
		Basketball	Hockey	Volleyball
Venue	A	28	23	23
	B	27	23	30
	C	25	26	22
	D	30	24	27

(a) Represent the choice of venues for each sport as a bipartite graph. (2 marks)



✓ 2 distinct sets
✓ joins
numbers not needed

(b) If venues B, C and D are chosen for basketball, hockey and volleyball respectively, calculate the total cost. (1 mark)

$27 + 26 + 27$

$= 80$

$= \$8000$ ✓

(c) Show use of the Hungarian algorithm to determine a suitable allocation of sports to venues in order to minimise the total cost, and state the minimum cost. (5 marks)

dummy column ✓

$$\begin{bmatrix} 3 & 0 & 1 & 0 \\ 2 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 \\ 5 & 0 & 5 & 0 \end{bmatrix}$$ ✓ rows ≠ lines

$$\begin{bmatrix} 2 & 0 & 0 & 0 \\ 1 & 0 & 7 & 0 \\ 0 & 4 & 0 & 1 \\ 4 & 1 & 4 & 0 \end{bmatrix}$$ ✓ rows = lines

Total 71

$\therefore \$7100$ ✓

$\therefore$ Basketball → C → 25
Hockey → B → 23 ✓
Volleyball → A → 23

End of questions